Enhancement of Gross Enrollment Ratio of Higher Education of West Bengal through e-Learning: A Fuzzy Delphi Forecasting Approach

Anirban Das, *Member, IACSIT,* Soumen Nandi, and Kajla Basu

***Abstract*—E-learning has now revolutionized the way of learning process across the world. Following the conventional systems of learning, the Gross Enrollment Ratio (GER) of West Bengal is not at all optimistic. To enhance the enrollments and diminish the drop outs in higher education in West Bengal, e-learning can be a better way out. In this paper an attempt has been made to forecast the increase in higher education enrollments and decrease in drop outs in higher education. Through Fuzzy Delphi Forecasting techniques views are collected from experts regarding the increase of higher education enrollments and decrease of drop outs through web enabled learning systems in comparison to the traditional learning systems.**

***Index Terms*—Drop outs, e-learning, enrollment, Fuzzy Delphi Forecasting, GER**

I. Introduction

Regarding higher education in West Bengal the most miserably significant information is that only 26.33% is the gross enrollment ratio. Though a couple of reasons like poverty, distance from home to higher education institutes, bad transportation system, expensive higher education, less seats in higher education institutes, mediocre students are not getting their subjects of interest, engagements with family occupation, being one of the earning members of the family etc.[1] are responsible for these wretched situation, e-learning can be a best way out to make the drop outs or going to be dropped out students interested in higher education.[2]

This paper narrates the forecasting through Fuzzy Delphi Method concerning the increase rate of higher education enrollments and decrease rate of drop outs by the starting of the year 2015.

II. Preliminaries

Fuzzy Delphi Method refers to a method of generalization of frequently used techniques for extended range forecasting. The extended range forecasting problems are like uncertain in nature; but hardly probabilistic. [3], [4] generally the responses or decisions provided by the experts are subjective in nature and based on their individual potentials. In Delphi Method, this is the key motive to make the decisions or responses represented through fuzzy numbers rather than crisp.

The Fuzzy Delphi Method (FDM) is applied here to adjust the evaluation of each expert to achieve the consensus condition of all experts consistent.[5] Through the following steps the Fuzzy Delphi Method is applied in this paper.

Step1:

We have considered the experts E_i provide the possible realization rating on forecasting of several contexts. The responses given by each and every expert are represented in the form of triangular fuzzy numbers i.e. TFN like:

$$A^{(i)} = (a_1^{(i)}, a_M^{(i)}, a_2^{(i)}), i = 1,2,3,4....., n$$

Step 2:

Average i.e. mean A_m of every $A^{(i)}$ is calculated, where average of all $a_1^{(i)}, a_M^{(i)}, a_2^{(i)}$ needs to get computed.

$$A_m = (m_1, m_M, m_2) = (\frac{1}{n}\sum_{i=1}^{n} a_1^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_M^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_2^{(i)})$$

For each and every expert E_i, the differences are calculated:

$$(m_1 - a_1^{(i)}, m_M - a_M^{(i)}, m_2 - a_2^{(i)}) =$$
$$(\frac{1}{n}\sum_{i=1}^{n} a_1^{(i)} - a_1^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_M^{(i)} - a_M^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_2^{(i)} - a_2^{(i)})$$

Now, the distances of $A^{(i)}$ from A_m are calculated through:

$$d(A^{(i)}, A_m)$$
$$= \frac{1}{2}\{\max(\left|m_1 - a_1^{(i)}\right|, \left|m_2 - a_2^{(i)}\right|) + \left|m_M - a_M^{(i)}\right|\}$$

The obtained resultant values are sent to the experts again for re-evaluation and assessment.

Step 3:

After re-evaluation and assessment each expert E_i provides revised value in the form of a triangular fuzzy

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Anirban Das is with the Department of Computer Applications, Dr. B. C. Roy Engineering College, Durgapur, West Bengal, India (e-mail:anirban-das@live.com, Phone: +919474623504)

Soumen Nandi is with Tata Consultancy Services, Mumbai, India (e-mail: soumen.tt@gmail.com, Phone: +918652276573).

Kajla Basu is with Department of Mathematics, National Institute of Technology, Durgapur, West Bengal, India (e-mail: kajla.basu@gmail.com, Phone: +919474532576).

number $B^{(i)} = (b_1^{(i)}, b_M^{(i)}, b_2^{(i)}), i = 1,2,3,4....., n$

This process starting with Step2 is repeated. The average and differences are calculated like:

$$(m_1 - b_1^{(i)}, m_M - b_M^{(i)}, m_2 - b_2^{(i)}) =$$
$$(\frac{1}{n}\sum_{i=1}^{n} b_1^{(i)} - b_1^{(i)}, \frac{1}{n}\sum_{i=1}^{n} b_M^{(i)} - b_M^{(i)}, \frac{1}{n}\sum_{i=1}^{n} b_2^{(i)} - b_2^{(i)})$$

Now, the distances of $B^{(i)}$ from B_m are calculated through:

$$d(B^{(i)}, B_m)$$
$$= \frac{1}{2}\{\max(\left|m_1 - b_1^{(i)}\right|, \left|m_2 - b_2^{(i)}\right|) + \left|m_M - b_M^{(i)}\right|\}$$

And, the process could be repeated again and again until two successive means A_m, B_m ,..... become reasonably close.

Step 4:

The similar procedure may be applied for reexamining or reassessing points of rating, if some important points arise due to various new circumstances.

III. Methodologies and Results

In accordance with the year wise view (starting from 2004 to 2010) of enrollments and drop out percentages in West Bengal [6] it is seen in Fig.1 that enrollments in higher education in West Bengal is always less than 50% of the total enrollments in secondary education and the range of drop outs in higher education is like 51.457% to 57.922%.

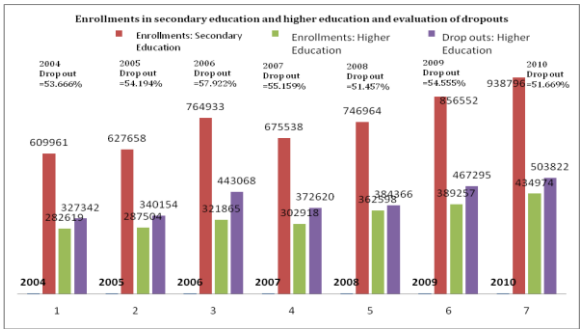

Fig. 1. Year wise enrollments in secondary education and higher education and evaluation of drop outs

From the experts of higher education ministry of West Bengal along with the headmasters and headmistresses of Bengali medium government schools a common survey questionnaire was canvassed and response data were collected concerning the forecasting of enhancements regarding enrollments of students in traditional system of learning and e-learning both. Besides the decrease of dropout rates are also asked to forecast by the starting of the year 2015.

The responses, we collected, from the experts are in the form of Triangular Fuzzy Number form where for a specific question three possible values were asked:

[For questions of enrollments only]
Lowest Enrollments
Most likely Enrollments
Highest Enrollments
And, [For questions of drop outs only]
Lowest Drop outs
Most likely Drop outs
Highest Drop outs

A. Fuzzy Delphi Approach in favor of traditional systems of higher education in West Bengal

Responses in Triangular Fuzzy Number Form to know increase of enrollments in traditional systems of learning in higher education by the starting of the year 2015 [in first stage of Fuzzy Delphi Method]

TABLE I: Fuzzy Responses for First Stage To Know Increase of Enrollments in Traditional Systems in West Bengal By The Starting of The Year 2015

Experts E_i	Initial values(in percentage) given by experts (first time responses) $a_1^{(i)}$	$a_M^{(i)}$	$a_2^{(i)}$
E_1	7	12	15
E_2	10	15	19
E_3	10	12	16
E_4	8	10	12
E_5	15	20	25
E_6	10	15	20
E_7	7	11	14
E_8	9	15	25
E_9	10	12	14
E_{10}	10	15	20
E_{11}	7	11	16
E_{12}	15	20	25
E_{13}	12	18	23
E_{14}	10	15	25
E_{15}	8	16	20
E_{16}	15	20	26
E_{17}	10	13	17
E_{18}	7	15	19
E_{19}	14	19	24
E_{20}	9	15	19
Average	10.15	14.95	19.7

$A_m = (m_1, m_M, m_2) = (10.15, 14.95, 19.7)$

Now in first stage the distances of each and every expert's responses from the average are calculated in TABLE II.

TABLE II: Distances of Each and Every Expert's Responses from the Average A_M

Distances of each expert's opinion from the average $A_m = (m_1, m_M, m_2) = (10.15, 14.95, 19.7)$				
Experts E_i	$m_1 - a_1^{(i)}$	$m_M - a_M^{(i)}$	$m_2 - a_2^{(i)}$	$d(A^{(i)} - A_m)$

E_1	3.15	2.95	4.7	**3.825**
E_2	0.15	-0.05	0.7	**0.375**
E_3	0.15	2.95	3.7	**3.325**
E_4	2.15	4.95	7.7	**6.325**
E_5	-4.85	-5.05	-5.3	**5.175**
E_6	0.15	-0.05	-0.3	**0.175**
E_7	3.15	3.95	5.7	**4.825**
E_8	1.15	-0.05	-5.3	**2.675**
E_9	0.15	2.95	5.7	**4.325**
E_{10}	0.15	-0.05	-0.3	**0.175**
E_{11}	3.15	3.95	3.7	**3.825**
E_{12}	-4.85	-5.05	-5.3	**5.175**
E_{13}	-1.85	-3.05	-3.3	**3.175**
E_{14}	0.15	-0.05	-5.3	**2.675**
E_{15}	2.15	-1.05	-0.3	**1.6**
E_{16}	-4.85	-5.05	-6.3	**5.675**
E_{17}	0.15	1.95	2.7	**2.325**
E_{18}	3.15	-0.05	0.7	**1.6**
E_{19}	-3.85	-4.05	-4.3	**4.175**
E_{20}	1.15	-0.05	0.7	**0.6**

By plotting the values of d in first stage we have the representation like:

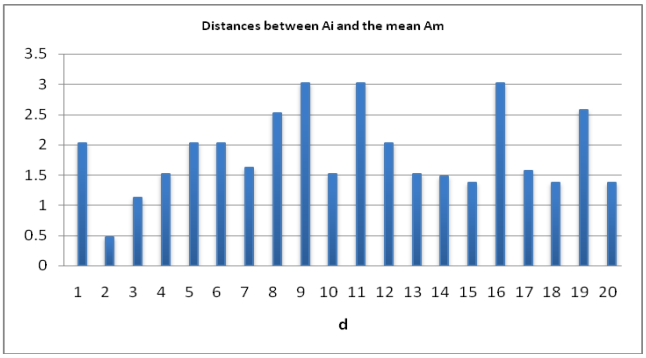

Fig. 2. Graphical representation of distances between $A^{(i)}$ to A_m

Responses in Triangular Fuzzy Number Form to know increase of enrollments in higher education in traditional systems of learning by the starting of the year 2015 [in second stage of Fuzzy Delphi Method]

TABLE III: FUZZY RESPONSES FOR SECOND STAGE TO KNOW INCREASE OF ENROLLMENTS IN TRADITIONAL SYSTEMS IN WEST BENGAL BY THE STARTING OF THE YEAR 2015

Experts	Initial values (in percentage) given by experts (second time responses)		
E_i	$b_1^{(i)}$	$b_M^{(i)}$	$b_2^{(i)}$
E_1	9	13	17
E_2	10	15	20
E_3	10	13	20
E_4	8	13	18
E_5	12	17	20
E_6	10	13	17
E_7	13	14	21
E_8	9	13	16
E_9	10	13	15
E_{10}	12	16	20
E_{11}	9	11	17
E_{12}	12	17	20
E_{13}	12	16	21
E_{14}	10	15	22
E_{15}	9	16	20
E_{16}	13	18	20
E_{17}	10	14	22
E_{18}	8	15	20
E_{19}	12	14	24
E_{20}	9	16	19
Average	10.35	14.6	19.45

$B_m=(m_1, m_M, m_2)$=(10.35, 14.6, 19.45)

Now in second stage the distances of each and every expert's responses from the average are calculated in TABLE IV.

TABLE IV: DISTANCES OF EACH AND EVERY EXPERT'S RESPONSES FROM THE AVERAGE B_M(IN SECOND STAGE)

Distances of each expert's opinion from the average $B_m=(m_1, m_M, m_2)$=(10.35, 14.6, 19.45)				
Experts E_i	$m_1 - b_1^{(i)}$	$m_M - b_M^{(i)}$	$m_2 - b_2^{(i)}$	$d(B^{(i)} - B_m)$
E_1	1.35	1.6	2.45	**2.025**
E_2	0.35	-0.4	-0.55	**0.475**
E_3	0.35	1.6	-0.55	**1.125**
E_4	2.35	1.6	1.45	**1.525**
E_5	-1.65	-2.4	-0.55	**2.025**
E_6	0.35	1.6	2.45	**2.025**
E_7	-2.65	0.6	-1.55	**1.625**
E_8	1.35	1.6	3.45	**2.525**
E_9	0.35	1.6	4.45	**3.025**
E_{10}	-1.65	-1.4	-0.55	**1.525**
E_{11}	1.35	3.6	2.45	**3.025**
E_{12}	-1.65	-2.4	-0.55	**2.025**
E_{13}	-1.65	-1.4	-1.55	**1.525**
E_{14}	0.35	-0.4	-2.55	**1.475**
E_{15}	1.35	-1.4	-0.55	**1.375**
E_{16}	-2.65	-3.4	-0.55	**3.025**
E_{17}	0.35	0.6	-2.55	**1.575**
E_{18}	2.35	-0.4	-0.55	**1.375**
E_{19}	-1.65	0.6	-4.55	**2.575**
E_{20}	1.35	-1.4	0.45	**1.375**

By plotting the values of d in second stage we have the representation like:

Here, B_m = (10.35, 14.6, 19.45) whereas Am= (10.15, 14.95, 19.7). So, B_m is very close to A_m. So, this result meets satisfactory in terms of acceptance. So, the iteration of same questions to same respondents is stopped now and the values of B_m is accepted.

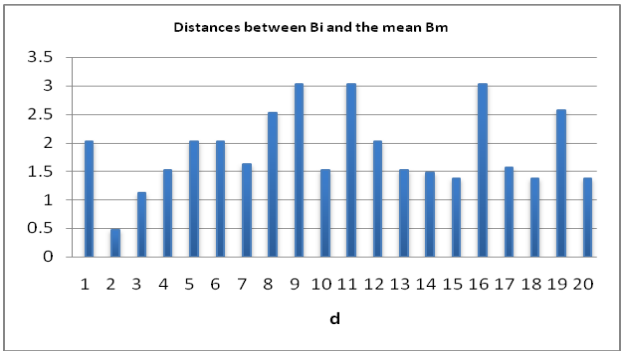

Fig. 3. Graphical representation of distances between $B^{(i)}$ to B_m

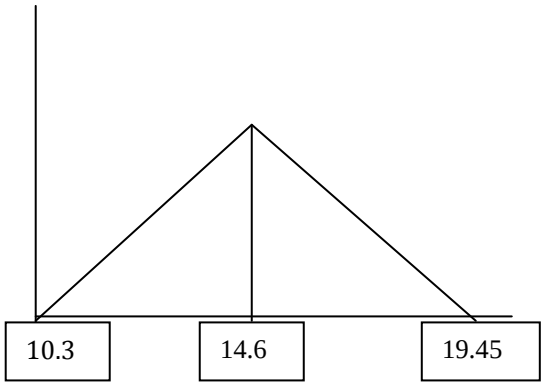

Fig. 4. Pictorial representation of the obtained triangular fuzzy responses.

Now for de-fuzzification we will consider the formula:

$$\{\sum_{i=1}^{20} b_1^{(i)}/20 + 4(\sum_{i=1}^{20} b_M^{(i)}/20) + \sum_{i=1}^{20} b_2^{(i)}/20\}/6$$

And it results: {10.35+4*(14.6)+19.45}/6 =14.7

So, from the above evaluations and calculations [Ref: TableI, Table II, Table III, Table IV, Fig. 2, Fig. 3, Fig. 4] it can be forecasted that the increase of enrollments of higher education through traditional systems of learning of West Bengal will increase 14.7% by the starting of the year 2015.

B. Fuzzy Delphi Approach in favor of e-learning based higher education in West Bengal

Responses in Triangular Fuzzy Number Form to know increase of enrollments in e-learning based systems of higher education by the starting of the year 2015 [in first stage of Fuzzy Delphi Method]

TABLE V: FUZZY RESPONSES FOR FIRST STAGE TO KNOW INCREASE OF ENROLLMENTS IN E-LEARNING BASED SYSTEMS OF HIGHER EDUCATION IN WEST BENGAL BY THE STARTING OF THE YEAR 2015

Experts E_i	Initial values(in percentage) given by experts (first time responses) $a_1^{(i)}$	$a_M^{(i)}$	$a_2^{(i)}$
E_1	10	15	20
E_2	13	20	22
E_3	11	15	20
E_4	9	10	19
E_5	17	22	28
E_6	15	20	22
E_7	11	15	20
E_8	13	18	22
E_9	15	19	25
E_{10}	12	17	21
E_{11}	11	15	22
E_{12}	19	22	25
E_{13}	15	20	25
E_{14}	12	17	20
E_{15}	12	17	22
E_{16}	19	25	28
E_{17}	14	20	23
E_{18}	11	19	24
E_{19}	15	25	29
E_{20}	15	25	30
Average	13.45	18.8	23.35

$A_m=(m_1, m_M, m_2)$=(13.45, 18.8, 23.35)

Now in first stage the distances of each and every expert's responses from the average are calculated in TABLE VI.

TABLE VI: DISTANCES OF EACH AND EVERY EXPERT'S RESPONSES FROM THE AVERAGE A_M

Distances of each expert's opinion from the average $A_m=(m_1, m_M, m_2)$=(13.45, 18.8, 23.35)				
Experts E_i	$m_1\_a_1^{(i)}$	$m_M\_a_M^{(i)}$	$m_2\_a_2^{(i)}$	$d(A^{(i)}\_A_m)$
E_1	3.45	3.8	3.35	**3.625**
E_2	0.45	-1.2	1.35	**1.275**
E_3	2.45	3.8	3.35	**3.575**
E_4	4.45	8.8	4.35	**6.625**
E_5	-3.55	-3.2	-4.65	**3.925**
E_6	-1.55	-1.2	1.35	**1.375**
E_7	2.45	3.8	3.35	**3.575**
E_8	0.45	0.8	1.35	**1.075**
E_9	-1.55	-0.2	-1.65	**0.925**
E_{10}	1.45	1.8	2.35	**2.075**
E_{11}	2.45	3.8	1.35	**3.125**
E_{12}	-5.55	-3.2	-1.65	**4.375**
E_{13}	-1.55	-1.2	-1.65	**1.425**
E_{14}	1.45	1.8	3.35	**2.575**
E_{15}	1.45	1.8	1.35	**1.625**
E_{16}	-5.55	-6.2	-4.65	**5.875**
E_{17}	-0.55	-1.2	0.35	**0.875**
E_{18}	2.45	-0.2	-0.65	**1.325**
E_{19}	-1.55	-6.2	-5.65	**5.925**
E_{20}	-1.55	-6.2	-6.65	**6.425**

By plotting the values of d in first stage we have the representation like:

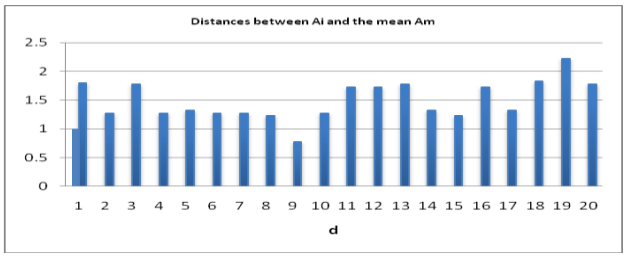

Fig. 5: Graphical representation of distances between $A^{(i)}$ to A_m

Responses in Triangular Fuzzy Number Form to know increase of enrollments in e-learning based systems of higher education in West Bengal by the starting of the year 2015 [in second stage of Fuzzy Delphi Method]

TABLE VII: FUZZY RESPONSES FOR SECOND STAGE TO KNOW INCREASE OF ENROLLMENTS IN HIGHER EDUCATION IN E-LEARNING BASED SYSTEMS IN WEST BENGAL BY THE STARTING OF THE YEAR 2015

Experts	Initial values (in percentage) given by experts (second time responses)		
E_i	$b_1^{(i)}$	$b_M^{(i)}$	$b_2^{(i)}$
E_1	12	17	21
E_2	12	18	20
E_3	13	15	23
E_4	12	16	22
E_5	15	22	26
E_6	13	18	20
E_7	13	17	21
E_8	15	18	20
E_9	14	21	27
E_{10}	11	16	22
E_{11}	10	17	20
E_{12}	17	21	26
E_{13}	16	22	26
E_{14}	13	17	21
E_{15}	14	18	27
E_{16}	17	23	26
E_{17}	15	22	26
E_{18}	12	20	27
E_{19}	14	20	26
E_{20}	17	21	26
Average	13.75	18.95	23.65

$B_m=(m_1, m_M, m_2)$=(13.75, 18.95, 23.65)

Now in second stage the distances of each and every expert's responses from the average are calculated in TABLE VIII.

TABLE VIII: DISTANCES OF EACH AND EVERY EXPERT'S RESPONSES FROM THE AVERAGE B_M (IN SECOND STAGE)

Distances of each expert's opinion from the average $B_m=(m_1, m_M, m_2)$=(13.75, 18.95, 23.65)				
Experts E_i	$m_1-b_1^{(i)}$	$m_M-b_M^{(i)}$	$m_2-b_2^{(i)}$	$d(B^{(i)}-B_m)$
E_1	1.75	1.95	2.65	**2.3**
E_2	1.75	0.95	3.65	**2.3**
E_3	0.75	3.95	0.65	**2.35**
E_4	1.75	2.95	1.65	**2.35**
E_5	-1.25	-3.05	-2.35	**2.7**
E_6	0.75	0.95	3.65	**2.3**
E_7	0.75	1.95	2.65	**2.3**
E_8	-1.25	0.95	3.65	**2.3**
E_9	-0.25	-2.05	-3.35	**2.7**
E_{10}	2.75	2.95	1.65	**2.85**
E_{11}	3.75	1.95	3.65	**2.85**
E_{12}	-3.25	-2.05	-2.35	**2.65**
E_{13}	-2.25	-3.05	-2.35	**2.7**
E_{14}	0.75	1.95	2.65	**2.3**
E_{15}	-0.25	0.95	-3.35	**2.15**
E_{16}	-3.25	-4.05	-2.35	**3.65**
E_{17}	-1.25	-3.05	-2.35	**2.7**
E_{18}	1.75	-1.05	-3.35	**2.2**
E_{19}	-0.25	-1.05	-2.35	**1.7**
E_{20}	-3.25	-2.05	-2.35	**2.65**

By plotting the values of d in second stage we have the representation like:

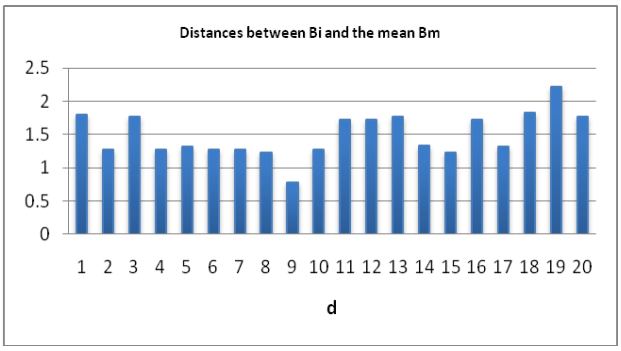

Fig. 6. Graphical representation of distances between $B^{(i)}$ to B_m

Here, B_m = (13.75, 18.95, 23.65) whereas Am= (13.45, 18.8, 23.35). So, B_m is very close to A_m. So, this result meets satisfactory in terms of acceptance. So, the iteration of same questions to same respondents is stopped now and the values of B_m is accepted.

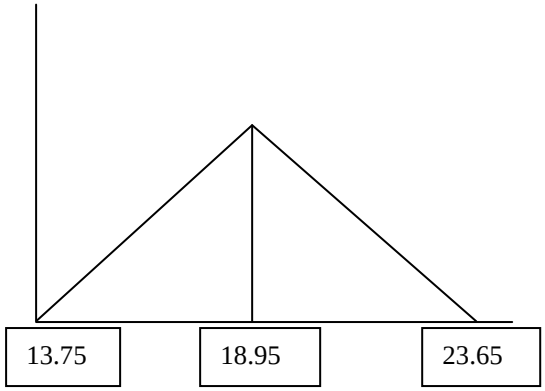

Fig. 7. Pictorial representation of the obtained triangular fuzzy responses.

Now for de-fuzzification we will consider the formula:

$$\{\sum_{i=1}^{20} b_1^{(i)}/20 + 4(\sum_{i=1}^{20} b_M^{(i)}/20) + \sum_{i=1}^{20} b_2^{(i)}/20\}/6$$

And it results: {13.75+4*(18.95)+23.65}/6 =18.866

So, from the above evaluations and calculations [Ref: TABLE V, TABLE VI, TABLE VII, TABLE VIII, Fig. 5, Fig. 6, Fig. 7 it can be forecasted that the increase of enrollments of higher education through e-learning systems of West Bengal will increase 18.866% by the starting of the year 2015.

C. Fuzzy Delphi Approach to calculate the decrease of dropouts in higher education in West Bengal

Responses in Triangular Fuzzy Number Form to know decrease of drop outs in higher education through e-learning systems by the starting of the year 2015 [in first stage of Fuzzy Delphi Method]

TABLE IX: FUZZY RESPONSES FOR FIRST STAGE TO KNOW DECREASE OF DROP OUTS IN HIGHER EDUCATION THROUGH E-LEARNING IN WEST BENGAL BY THE STARTING OF THE YEAR 2015

Experts E_i	Initial values(in percentage) given by experts (first time responses) $a_1^{(i)}$	$a_M^{(i)}$	$a_2^{(i)}$
E_1	1	3	5
E_2	2	5	7
E_3	1	3	5
E_4	3	7	9
E_5	4	6	9
E_6	1	3	5
E_7	2	5	6
E_8	2	5	7
E_9	1	4	7
E_{10}	3	6	8
E_{11}	2	7	9
E_{12}	4	8	10
E_{13}	5	8	10
E_{14}	5	7	9
E_{15}	6	9	11
E_{16}	5	8	10
E_{17}	4	7	9
E_{18}	5	8	10
E_{19}	4	7	9
E_{20}	3	5	8
Average	3.15	6.05	8.15

$A_m=(m_1, m_M, m_2)$=(3.15, 6.05, 8.15)

Now in first stage the distances of each and every expert's responses from the average are calculated in TABLE X.

TABLE X: DISTANCES OF EACH AND EVERY EXPERT'S RESPONSES FROM THE AVERAGE A_m

Distances of each expert's opinion from the average $A_m=(m_1, m_M, m_2)$=(3.15, 6.05, 8.15)				
Experts E_i	$m_1-a_1^{(i)}$	$m_M-a_M^{(i)}$	$m_2-a_2^{(i)}$	$d(A^{(i)}-A_m)$
E_1	2.15	3.05	3.15	3.1
E_2	1.15	1.05	1.15	1.1
E_3	2.15	3.05	3.15	3.1
E_4	0.15	-0.95	-0.85	0.9
E_5	-0.85	0.05	-0.85	0.45
E_6	2.15	3.05	3.15	3.1
E_7	1.15	1.05	2.15	1.6
E_8	1.15	1.05	1.15	1.1
E_9	2.15	2.05	1.15	2.1
E_{10}	0.15	0.05	0.15	0.1
E_{11}	1.15	-0.95	-0.85	1.05
E_{12}	-0.85	-1.95	-1.85	1.9
E_{13}	-1.85	-1.95	-1.85	1.9
E_{14}	-1.85	-0.95	-0.85	1.4
E_{15}	-2.85	-2.95	-2.85	2.9
E_{16}	-1.85	-1.95	-1.85	2.4
E_{17}	-0.85	-0.95	-0.85	0.9
E_{18}	-1.85	-1.95	-1.85	1.9
E_{19}	-0.85	-0.95	-0.85	0.9
E_{20}	0.15	1.05	0.15	0.6

TABLE XI: FUZZY RESPONSES FOR SECOND STAGE TO KNOW DECREASE OF DROP OUTS IN HIGHER EDUCATION THROUGH E-LEARNING SYSTEMS IN WEST BENGAL BY THE STARTING OF THE YEAR 2015

Experts E_i	Initial values (in percentage) given by experts (second time responses) $b_1^{(i)}$	$b_M^{(i)}$	$b_2^{(i)}$
E_1	2	4	7
E_2	2	5	7
E_3	2	5	6
E_4	3	5	7
E_5	3	6	11
E_6	2	5	7
E_7	4	5	7
E_8	2	7	10
E_9	2	6	7
E_{10}	3	5	7
E_{11}	3	8	10
E_{12}	5	8	10
E_{13}	2	4	7
E_{14}	5	7	8
E_{15}	4	8	9
E_{16}	5	7	11
E_{17}	4	6	11
E_{18}	5	8	10
E_{19}	4	9	10
E_{20}	3	4	7
Average	3.25	6.1	8.45

$B_m=(m_1, m_M, m_2)$=(3.25, 6.1, 8.45)

Now in second stage the distances of each and every expert's responses from the average are calculated in TABLE XII.

TABLE XII: DISTANCES OF EACH AND EVERY EXPERT'S RESPONSES FROM THE AVERAGE B_M (IN SECOND STAGE)

Distances of each expert's opinion from the average $B_m=(m_1, m_M, m_2)$=(3.25, 6.1, 8.45)				
Experts E_i	$m_1-b_1^{(i)}$	$m_M-b_M^{(i)}$	$m_2-b_2^{(i)}$	$d(B^{(i)}-B_m)$
E_1	1.25	2.1	1.45	**1.8**
E_2	1.25	1.1	1.45	**1.275**
E_3	1.25	1.1	2.45	**1.775**
E_4	0.25	1.1	1.45	**1.275**
E_5	0.25	0.1	-2.55	**1.325**
E_6	1.25	1.1	1.45	**1.275**
E_7	-0.75	1.1	1.45	**1.275**
E_8	1.25	-0.9	-1.55	**1.225**
E_9	1.25	0.1	1.45	**0.775**
E_{10}	0.25	1.1	1.45	**1.275**
E_{11}	0.25	-1.9	-1.55	**1.725**

E_{12}	-1.75	-1.9	-1.55	**1.725**
E_{13}	1.25	2.1	1.45	**1.775**
E_{14}	-1.75	-0.9	0.45	**1.3275**
E_{15}	-0.75	-1.9	-0.55	**1.225**
E_{16}	-1.75	-0.9	-2.55	**1.725**
E_{17}	-0.75	0.1	-2.55	**1.325**
E_{18}	-1.75	-1.9	-1.55	**1.825**
E_{19}	-0.75	-2.9	-1.55	**2.225**
E_{20}	0.25	2.1	1.45	**1.775**

By plotting the values of d in first stage we have the representation like:

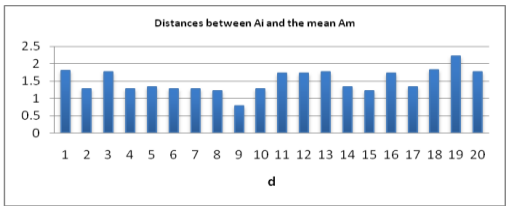

Fig. 8. Graphical representation of distances between $A^{(i)}$ to A_m

Responses in Triangular Fuzzy Number Form to know decrease of drop outs in higher education through e-learning systems in West Bengal by the starting of the year 2015 [in second stage of Fuzzy Delphi Method]

By plotting the values of d in second stage we have the representation like:

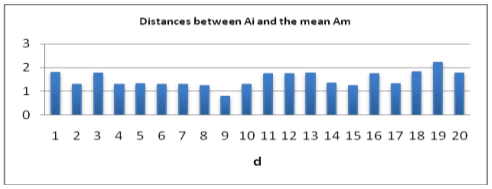

Fig. 9. Graphical representation of distances between $B^{(i)}$ to B_m

Here, B_m =(3.25, 6.1, 8.45) whereas A_m=(3.15, 6.05, 8.15). So, B_m is very close to A_m. So, this result meets satisfactory in terms of acceptance. So, the iteration of same questions to same respondents is stopped now and the values of B_m is accepted.

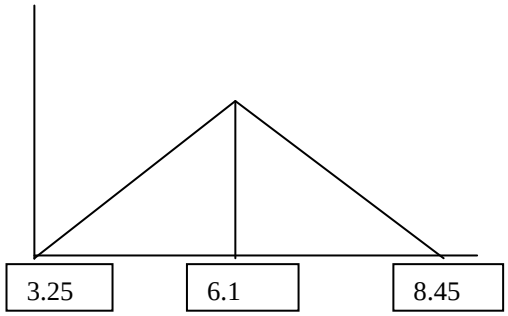

Fig. 10. Pictorial representation of the obtained triangular fuzzy responses.

Now for de-fuzzification we will consider the formula:

$$\{\sum_{i=1}^{20} b_1^{(i)}/20 + 4(\sum_{i=1}^{20} b_M^{(i)}/20) + \sum_{i=1}^{20} b_2^{(i)}/20\}/6$$

And it results: {3.25+4*(6.1)+8.45}/6 =6.016

So, from the above evaluations and calculations [Ref: Table IX, Table X, Table XI, Table XII, Fig. 8, Fig. 9, Fig. 10] it can be forecasted that the drop outs of higher education through e-learning systems of West Bengal will decrease 6.016% by the starting of the year 2015.

IV. Conclusion

Following the detailed analysis of forecasting through Fuzzy Delphi Method, it can be concluded that by the starting of the year 2015, the increase of enrollments of higher education through traditional systems of learning of West Bengal will increase 14.7% whereas increase of enrollments of higher education through e-learning systems of West Bengal will increase 18.866% and the drop outs of higher education through e-learning systems of West Bengal will decrease 6.016%. So, the gain of enrollments through e-learning will be 4.166% and also the gain in decreasing drop outs will be 6.016%, which leads to increase the overall GER of higher education in West Bengal by 2015.

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Anirban Das is pursuing his research work in the Department of Mathematics from National Institute of Technology, Durgapur, India. He is a member of IAENG and IACSIT. He is having around 4 years of work experience including academics and industry. He started his career with HCL Technologies as a software Engineer. He is Microsoft Certified Technology Specialist (MCTS) in Microsoft Office Sharepoint. Server 2007 Presently he is the Assistant Professor in Computer Applications department in Dr. B. C. Roy Engineering College, Durgapur, India. He has around 10 research papers in various eminent journal and conferences. Mr. Das has interests in Statistical Analysis, e-commerce, e-learning and computer applications as domain of research.

Soumen Nandi is Assistant Systems Engineer - Trainee in Tata Consultancy Services, Mumbai. He passed M. Tech in Operations Research from National Institute of Technology, Durgapur, India in 2011. He passed B. Tech in Textile Technology from West Bengal University of Technology, Kolkata, India. Mr. Nandi has interests in Statistical Analysis, Operations Research, e-learning, forecasting, Fuzzy Mathematics etc as domain of research.

Kajla Basu is as Associate Professor in department of Mathematics in National Institute of Technology, Durgapur, India. She passed M. Sc. in Mathematics with national scholarship along with C.S.I.R, J.R.F and SRF (for 5 years) and Ph.D degree from IIT, Kharagpur. She is having around 15 years of work experience at National Institute of Technology, Durgapur. She has around 13 research papers in various eminent journals and conferences. Prof. Basu is a life member of ORSI and her domain of research interests are in Statistical Analysis, Operations Research and Fuzzy Mathematics